Abstract

In this dissertation, both qualitative and numerical analysis for an optimization problem is performed for a feedback control law applied to a class of nonlinear reaction-diffusion processes. A finite number of control and measurement devices target their actions inside the process domain. The measurement devices collect data on the process evolution, while the control devices obtain those data and activate an appropriate reaction. The aim of this control system is to keep the process evolution close to a user-defined reference state. The above optimization problem consists in choosing geometrical targeting of the control and measurement devices actions according to a suitable optimality criterion.

Such an idea of the closed-loop control of reaction-diffusion processes is implemented by a system of equations with a semilinear PDE coupled to several nonlinear ODEs. The cost functional utilized for a precise definition of the announced problem of optimal targeting is constructed as an integral of the difference between the process and reference states.

The present work is divided into two main parts. The first of them focuses on analysis of the PDE-ODE model under consideration. The second one concerns the problem of optimal targeting, exploiting some of the results of the first part.

In the analysis of the PDE-ODE model we focus on questions concerning existence, uniqueness and stability of solutions as well as on the efficiency of the closed-loop control mechanism implemented there. By efficiency we mean here an ability of moving the process close to the reference state. The existence, uniqueness and stability proofs are provided. The efficiency of the closed-loop control is validated by results of numerical simulations for the investigated PDE-ODE model. The numerical results suggest that the efficiency of the considered closed-loop control depends on changes of the model parameters. Moreover, the long-time behavior visible in the subject simulations also is examined. In all simulations, the process appeared to tend to some time-invariant state, after sufficiently long time. In some cases, that time-invariant state seemed to be, at some rate, independent of the initial condition of the PDE-ODE model.

In the part on the optimal targeting problem, we first focus on analytical questions. We prove there the existence of minimizers and characterize the differential of the cost functional too. Then, we describe numerical optimization experiments, utilizing three gradient optimization algorithms (the steepest descent and two variants of the nonlinear conjugate gradient) and compare their performance. Here, the aforementioned characterization of the cost functional differential is used to implement the formula for the gradient. The results show how the performance of the optimization algorithms varies with changes of the parameters entering the cost functional. It is also shown that modifications of the subject parameters can result in independence of the optimization output on the initial condition of the PDE-ODE model.